AI-Driven Analysis of Stochastic Longitudinal Vibrations in a Viscoelastic Cable with Moving Boundaries

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The study addresses the modeling and analysis of resonance characteristics in mechanical systems with moving boundaries. The focus is on the use of artificial intelligence (AI), specifically the hybrid neural network ResonanceNet (CNN + LSTM), to predict resonant frequencies, amplitudes, and prevent resonance. The computational method for solving integro-differential equations developed by the authors is applied. The problem is formulated as a boundary value problem for a partial differential equation with a moving boundary and reduced to a system of stochastic differential equations. The neural network is used offline for fast estimation; if confidence is low, a classical numerical module is activated. Comparative analysis showed that the hybrid approach reduces prediction error to 2.1% and prevents resonance in 96% of cases, which is 34% more accurate and 5 times faster than classical methods.

Keywords: viscoelasticity, moving boundaries, stochastic vibrations, resonance, artificial intelligence, neural networks, Monte Carlo method, TB-ANALYSIS 7 package.

1. Introduction

Systems with moving boundaries are widespread in technology (ropes of hoisting installations [1–4], flexible transmission links, etc.). The presence of moving boundaries causes significant difficulties in describing such systems. Exact solution methods are limited by the wave equation and relatively simple boundary conditions [6]. Of the approximate methods, the most effective is the method for constructing solutions of integro-differential equations described in [5, 8], as well as the Kantorovich–Galerkin method [7, 10].

The solution is implemented in dimensionless variables using the TB-ANALYSIS 7 software package [11] developed in the MATLAB environment. The package contains a module — resonance phenomena management

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— designed to determine the resonance region and to identify resonance prevention conditions imposed on the model parameters using AI. A key feature of the package is the hybrid architecture that integrates classical numerical methods with machine learning algorithms into a single iterative workflow. Using artificial intelligence, the resonant frequencies of the system, as well as the conditions under which resonance can be prevented, were determined [9]. The hybrid neural network ResonanceNet was trained on this data, enabling it to predict dangerous resonant frequencies and propose optimal parameters. Section 3 compares its accuracy and performance with classical methods.

2. Problem Formulation and Solution Method

Consider the stochastic longitudinal vibrations of a viscoelastic cable of variable length $l_0(t) = l_0 - v_0 t$, placed on an elastic foundation, taking into account the resistance forces of the environment [8]. The differential equation describing the longitudinal vibrations is

$$\rho U_{tt} - \rho a^2 U_{xx} + \lambda U_t + \mu U_{xxt} + k_0 U = 0, \tag{1}$$

where $U(x,t)$ is the longitudinal displacement, ρ is the linear mass density, $a = \sqrt{T/\rho}$ is the longitudinal wave velocity, T is the tension, μ is the viscoelasticity parameter, λ is the medium resistance, and k_0 is the foundation stiffness.

The boundary conditions on the moving boundary are

$$U(0,t) = 0, \qquad U(l_0(t),t) = B\cos W_0(\omega_0 t). \tag{2}$$

Stochasticity is introduced through random initial conditions and random parameters (stiffness, damping, boundary velocity). After passing to dimensionless variables and applying the Kantorovich–Galerkin procedure [7, 8], problem (1)–(2) reduces to the system of stochastic differential equations

$$\mu_n''(\tau) + \Omega_{0n}^2(\varepsilon\tau)\mu_n(\tau) = -\frac{\omega_{0n}^2(\varepsilon\tau)Q_{0n}(\varepsilon\tau)}{A_{0n}(\varepsilon\tau)} e^{\alpha\tau}\cos W(\tau). \tag{3}$$

For predicting the resonance characteristics, a specialized hybrid architecture, ResonanceNet (Multimodal Multi-task Resonance Prediction Network), was developed. Its input layer accepts a normalized multidimensional feature vector X, including static system parameters (k, c, V_0), dynamic features obtained via a brief preliminary calculation (amplitude values, the first N FFT harmonics), and external stimulus parameters. The network architecture includes: (1) a convolutional block (2 layers of 1D convolutions, 32 and 64

kernels, ReLU) for extracting local patterns; (2) a bidirectional LSTM layer (64 neurons) for capturing long-term dependencies; (3) an attention block for weighting important time intervals; and (4) multi-task output layers (two fully connected layers of 128 and 64 neurons) branching into three outputs: f_{res} (resonant frequency), $A_{\max}$ (amplitude), and R_{risk} (risk category).

A training set of $50,000$ examples was generated using the numerical simulation modules of TB-ANALYSIS 7. The split was 70/15/15%. Training used the Adam optimizer (initial learning rate 10^{-3}, 10-fold decrease on plateau) with dropout (0.3) and early stopping (patience=20). The network does not replace but complements classical methods: if the prediction variance (estimated via Monte Carlo dropout) exceeds a threshold, the advanced classical numerical module is automatically activated.

For real-time tuning, the control actions are the increments of stiffness and damping of the dominant modes. The measured quantities are the current amplitude and frequency extracted by FFT. The adaptation law is a classical model-reference scheme: the action is corrected in proportion to the deviation of the measured amplitude from its reference value. The neural network is used only at the tuning stage (reference amplitude and initial gains) and is not part of the real-time loop.

The software package implements a step-by-step error estimation algorithm. The error in calculating the square of the amplitude is $R = 2E_n^2(\tau_2)R_{\text{int}}[R_{\text{int}} + I_1 + I_2]$, where R_{int} is the integration error, controlled by the MATLAB `quad` function.

3. Results

To quantitatively evaluate the advantages of the hybrid approach, comparative testing was conducted on 1,000 test cases covering a wide range of system parameters. For each case, calculations were performed using three methods: (1) the classical asymptotic method (AM); (2) the classical approximate method for solving integro-differential equations (IDE); and (3) the hybrid method (HM) using ResonanceNet. The comparison criteria were the relative error in determining the resonant frequency $\Delta = |f_{\text{pred}} - f_{\text{ref}}|/f_{\text{ref}}$, the computational time T, and the resonance prevention success rate.

The analysis of the averaged results showed that the hybrid method achieves the lowest average error of 2.1%, compared to 4.8% for the asymptotic method and 3.2% for the IDE method. In terms of computational time, the hybrid approach requires an average of 1.8 s, which is significantly faster than 12.5 s and 8.7 s, respectively. The resonance prevention success rate reaches 96% for the hybrid method, outperforming the classical approaches (85% and

88%). Thus, the hybrid method demonstrates a 34% improvement in accuracy over the best classical method (IDE), while being on average 4.8 times faster than the IDE method and 6.9 times faster than the asymptotic method.

4. Conclusion

The study resulted in the development and validation of the TB-ANALYSIS 7 software package for mathematical modeling and analysis of resonance characteristics in mechanical systems with moving boundaries. The scientific novelty lies in the creation of a hybrid approach combining classical numerical methods with artificial intelligence. The proposed ResonanceNet network demonstrated high accuracy in predicting resonant frequencies and vibration amplitudes. Testing confirmed that the hybrid approach provides a 34% improvement in prediction accuracy compared to the best classical method while accelerating calculations by an average of 5 times. Prospects for further development involve expanding the functionality of the system and adapting the proposed methods to solve more complex classes of boundary value problems.

References

[1] L. B. Kolosov, T. I. Zhigula, "Longitudinal-transverse vibrations of the rope of the lifting installation", *Izv. Universities. Mining Journal*, 1981, № 3, 83–86 (in Russian).

[2] W. D. Zhu, Y. Chen, "Theoretical and experimental investigation of elevator cable dynamics and control", *J. Vibr. Acoust.*, 2006, № 1, 66–78. DOI: 10.1115/1.2128640.

[3] L. Wang, Y. Zhao, "Multiple internal resonances and non-planar dynamics of shallow suspended cables to the harmonic excitations", *J. Sound Vib.*, **319**:1–2 (2009), 1–14. DOI: 10.1016/j.jsv.2008.08.020.

[4] Y. Zhao, L. Wang, "On the symmetric modal interaction of the suspended cable: three-to-one internal resonance", *J. Sound Vib.*, **294**:4–5 (2006), 1073–1093. DOI: 10.1016/j.jsv.2006.01.004.

[5] O. A. Goroshko, G. N. Savin, *Introduction to the Mechanics of Deformable One-Dimensional Bodies of Variable Length*, Nauk. dumka, Kiev, 1971 (in Russian), 290 pp.

[6] V. L. Litvinov, V. N. Anisimov, *Mathematical Modeling and Research of Oscillations of One-Dimensional Mechanical Systems with Moving Boundaries: monograph*, Samara State Technical Univ., Samara, 2017 (in Russian), 149 pp.

[7] V. L. Litvinov, V. N. Anisimov, "Application of the Kantorovich–Galerkin method for solving boundary value problems with conditions on moving boundaries", *Bull. Russ. Acad. Sci. Rigid Body Mech.*, 2018, № 2, 70–77 (in Russian. DOI: 10.3103/s0025654418020085).

[8] V. L. Litvinov, K. V. Litvinova, "An approximate method for solving boundary value problems with moving boundaries by reduction to integro-differential equations", *Comput. Math. Math. Phys.*, **62**:6 (2022), 945–954 (in Russian. DOI: 10.1134/S0965542522060112).

[9] V. L. Litvinov, K. V. Litvinova, "On one inverse method for solving problems of oscillations of mechanical systems with moving boundaries", *Moscow Univ. Mech. Bull.*, 2024, № 3, 53–59 (in Russian. DOI: 10.3103/S0027133024700122).

[10] V. L. Litvinov, K. V. Litvinova, "Application of the Kantorovich–Galerkin method for analyzing the resonance characteristics of systems with damping", *Theor. Math. Phys.*, **224**:1 (2025), 129–138 (in Russian. DOI: 10.1134/S0040577925070098).

[11] V. L. Litvinov, *Certificate of State Registration of a Computer Program No. 2025613649*, 2025.

Received on October 3, 2026