Necessary and sufficient conditions for the existence of an image with a given code [1]

D.V. Alekseev[2]

The article introduces an image encoding function which is invariant with respect to affine transform. The properties of the encoding funciton are investigated. Necessary and sufficient conditions are found for a given set of numbers to be a code of nonsingulari image.

Keywords: image code, image encoding, affine equivalence.

Introduction

Recognition tasks often require some image encoding. One of the most commonly used image codes is just the coordinates of its points. This encoding is not invariant under geometric transformations, such as translation, rotation, stretching. Despite that the images obtained by such transformations are considered to be equivalent. In addition, that encoding implies fixing some external (to an image) coordinate system.

An affinity invariant image coding was introduced in the papers [4]-[5]. It was shown that the the codes equality of two images is a necessary and sufficient for them to be affine equivalent. This work introduces a modified coding function and researches the properties of that coding function. As a result the necessary and sufficient conditions are derived for an existence of image producing the given code.

The necessary and sufficient conditions for the existence of a three-dimensional image with two given planar projections were derived in [2].

In [5] an affine invariant coding function ρ was introduced: $\rho_{ijk,lmp} = \frac{S(\triangle a_i a_j a_k)}{S(\triangle a_l a_m a_p)}$ where $S(abc)$ stands for the area of the triangle abc. Thus, a set of n points is encoded by $(C_n^3)^2$ real numbers. Obviously, this code is redundant. This work examines the degree of its redundancy. In case of modified encoding the explicit conditions were derived that an arbitrary list of real numbers is the code of some image. For the original encoding function, the respective (implicit) conditions are also given.

In this paper we consider a modified coding function $r_{ijk,lmp} = \frac{S'(\triangle a_i a_j a_k)}{S'(\triangle a_l a_m a_p)}$, where S' stands for the oriented area, i.e. area with a $\pm$ sign depending on the triangle orientation.

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[1] Originally published in *Intellektualnye Sistemy. Teoriya i prilogeniya* (2020) **24**, No. 2, 55-66 (in Russian).

[2] Alekseev Dmitriy Vladimirovich — Candidate of Physical and Matematical Sciences, senior staff scientist, Lomonosov Moscow State University, Faculty of Mechanics and Mathematics, Problems of Theorecical Cybernetics Lab, e-mail: dvalex@rambler.ru

The rest of the paper is organized in the following way: The basic concepts and notation are introduced in section 1. The properties of the image code matrix are researched in section 2. The main result is formulated and proven in section 3. Section 4 is a conclusion.

1. Concepts and notation

Let S' be the oriented triangle area, i.e. $S'(\triangle abc) = S(\triangle abc)$ for the positive triangle orientation and $S'(\triangle abc) = -S(\triangle abc)$ for the negative one. The triangle orientation is considered to be positive when the triangle vertices are traversed in counterclockwise order and negative otherwise.

Consider the points $a_1, ..., a_n$ on a plane, let call the set $A = \{a_1, ..., a_n\}$ an image. An image is called emphdegenerate if all points lie on one straight line and *non-degenerate* otherwise. Fix some (Euclidean) coordinate system, the coordinates of the point a_i will be denoted as $X(a_i)$ and $Y(a_i)$. In the following, for convenience, individual indices will be denoted by lowercase Latin letters.

Let call *multi-index*, a vector comprising three indices. The multi-indices will be denoted later in the text by lowercase Greek letters $\alpha, \beta, \gamma, ...$. The multi-index components will be denoted by $\alpha = [\alpha(1), \alpha(2), \alpha(3)]$. The triangle [1] with the respective vertex indices will be denoted as $\triangle_\alpha = \triangle a_{\alpha(1)} a_{\ alpha(2)} a_{\alpha(3)}$.

Let call multi-indices α and α' *equivalent* if and only if the permutation $\begin{pmatrix} \alpha(1) & \alpha(2) & \alpha(3) \\ \alpha'(1) & \alpha'(2) & \alpha'(3) \end{pmatrix} \in S_3$ is even and denote it $\alpha \simeq \alpha'$. Let call the multi-index conjugate to α and denote it $\bar{\alpha}$ if the permutation $\begin{pmatrix} \alpha(1) & \alpha(2) & \alpha(3) \\ \bar{\alpha}(1) & \bar{\alpha}(2) & \bar{\alpha}(3) \end{pmatrix} \in S_3$ is odd. Obviously the triangles with equivalent multi-indices have the same oriented area, and the triangles with conjugate multi-indices have areas with the same absolute values and different signs. Later we do not distinguish between equivalent multi-indices i.e. regard them as the same multi-index. The same will be applies to the respective triangles.

In total, there are C_n^3 different unoriented triangles with vertices from $A = \{a_1, ..., a_n\}$, and respectively $N = 2 \cdot C_n^3$ oriented ones.

Let enumerate all multi-indices (and the respective triangles): $\alpha_1, ..., \alpha_N$. Let $\mathcal{A} = \{\alpha_1, ..., \alpha_N\}$ be the set of all multi-indices, and $E : \alpha_i \mapsto i$ be the respective enumeration function.

Consider the following set of fractions: $r_{ijk,lmp} = \frac{S'(\triangle a_i a_j a_k)}{S'(\triangle a_l a_m a_p)}$. If triangle $\triangle a_l a_m a_p$ is degenerate, i.e. $S'(\triangle a_l a_m a_p) = 0$, then use formal notation

[1] We will also use this notation in the case when the triangle is degenerate.

$r_{ijk,lmp} = \infty$. Let call the set the *code* for image $\{a_1, ..., a_n\}$. Similar encoding procedure was proposed in [1].

Definition 1. Consider $N \times N$ matrix $R = (r_{ij})$ with the elements $r_{ij} = r_{\alpha_i,\alpha_j} = \frac{S'(\triangle_{\alpha_i})}{S'(\triangle_{\alpha_j}}$. Thus, the elements of the image code are arranged in a square table, in which the rows and columns are enumerated by multi-indices (triangles). Let's call R the image *code matrix*.

Note 1. Leater the notation $r_{\ alpha\ beta} = R_{E(\alpha)E(\beta)}$ will be, i.e. rows and columns of the code matrix can also be indexed with multi-indices.

Example 1. Consider a trapezoid $a_1a_2a_3a_4$, with bases a_1a_2 and a_4a_3 such that $|a_1a_2| : |a_4a_3| = 1 : 2$ (see fig. 1). Let enumerate multi-indices as in

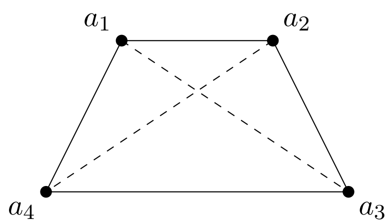

Fig. 1. Example 1.

table 1. Notice that the last four multi-indices are conjugate to the first four, so it is sufficient to construct only the part of the image code matrix corresponding to the first 4 rows and columns. The submatrix is the following:
$R_4 = \begin{pmatrix} 1 & 1 & 1/2 & 1/2 \\ 1 & 1 & 1/2 & 1/2 \\ 2 & 2 & 1 & 1 \\ 2 & 2 & 1 & 1 \end{pmatrix}$. The complete code matrix has the following form:
$R = \begin{pmatrix} R_4 & -R_4 \\ -R_4 & R_4 \end{pmatrix}$.

2. The properties of a code matrix

1) $r_{\alpha\alpha} = 1$ или ∞, for all $\alpha \in \mathcal{A}$ (**reflexivity**).

2) For all $\alpha, \beta \in \mathcal{A}$ such that $r_{\alpha\beta} \notin \{0, \infty\}$ holds $r_{\beta\alpha} = r_{\alpha\beta}^{(-1)}$ (**anti-symmetry**[2]).

3) For all $\alpha, \beta, \gamma \in \mathcal{A}$ such that $r_{\alpha\beta}, r_{\beta\gamma} \notin \{0, \infty\}$ holds $r_{\alpha\gamma} = r_{\alpha\beta} \cdot r_{\beta\gamma}$ (**transitivity**).

[2] If $r_{\alpha\beta} = 0$ then $r_{\beta\alpha} = \infty$. The converse is generally not true.

i	α_i
1	$1,2,3$
2	$1,2,4$
3	$1,3,4$
4	$2,3,4$
5	$1,3,2$
6	$1,4,2$
7	$1,4,3$
8	$2,4,3$

Table 1. Multi-index table

4) Let $\pi, \sigma \in S_3$ and $\alpha, \beta \in \mathcal{A}$. Let multi-indices $\alpha' = \pi(\alpha)$ and $\beta' = \sigma(\beta)$ are the results of permutations π and σ applied to multi-indices α and β, respectively $\alpha' = [\alpha(\pi(1)), \alpha(\pi(2)), \alpha(\pi(3))]$ и $\beta' = [\beta(\sigma(1)), \beta(\sigma(2)), \beta(\sigma(3))]$. Then either $r_{\alpha'\beta'} = (-1)^\pi \cdot (-1)^\sigma \cdot r_{\alpha\beta}$, or $r_{\alpha\beta} = \infty = r_{\alpha'\beta'}$ (**consistency with index permutations**).

5) Let $i_1, i_2, i_3, i_4 \in \{1, \ldots, N\}$, $\alpha_1 = [i_2, i_3, i_4]$, $\alpha_2 = [i_3, i_4, i_1]$, $\alpha_3 = [i_4, i_1, i_2]$ и $\alpha_4 = [i_1, i_2, i_3]$. Then for any $\beta \in \mathcal{A}$ the equality $r_{\alpha_1\beta} + r_{\alpha_3\beta} = r_{\alpha_2\beta} + r_{\alpha_4\beta}$ holds[3] (**additivity**).

Properties 1-3 are obvious. Property 4 follows from the change in the oriented area sign at permutations of vertices. To prove property 5 we calculate the area of a quadrilateral $a_{i_1}a_{i_2}a_{i_3}a_{i_4}$ (see. fig. 2) by two ways:

$$S'(a_{i_1}a_{i_2}a_{i_3}a_{i_4}) = S'(\triangle a_{i_2}a_{i_3}a_{i_4}) + S'(\triangle a_{i_4}a_{i_1}a_{i_2}) = \\ = S'(\triangle a_{i_3}a_{i_4}a_{i_1}) + S'(\triangle a_{i_1}a_{i_2}a_{i_3}).$$

Divide the equality by $S'(\triangle_\beta)$ and get the property 5.

It is natural to ask: are these conditions sufficient for an arbitrary matrix to be the code of some image? A counterexample below will show that this is not true.

Let prove the following helper lemma:

Lemma 1. *Consider non-degenerate triangle $\triangle_\beta$ and denote $\rho_\alpha = r_{\alpha\beta}$, $\alpha \in \mathcal{A}$. Fix an euclidean coordinates on a plane. Then there exist an affine transform F such that images of $a_i, i = 1, ..., n$ i.e. $c_i = F(a_i)$, have coordinates $X(c_i) = \rho_{i,\beta(3),\beta(1)}$, $Y(c_i) = \rho_{i,\beta(1),\beta(2)}$.*

Proof. Assume without lost of generality that $\beta = [1, 2, 3]$. There exists one and only one affine transform A such that $a_1 \mapsto c_1(0, 0)$, $a_2 \mapsto c_2(1, 0)$

[3]Consider the equation formally, as $\infty + \infty = \infty + \infty$, when the denominator is zero.

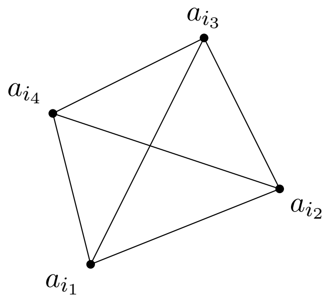

Fig. 2. Area Additivity: $S'_{234} + S'_{124} = S'_{123} + S'_{134}$.

and $a_3 \mapsto c_3(0,1)$. Let (x_i, y_i) be the coordinates of $c_i = A(a_i)$, $i = 1,2,3$. Then $S'(\triangle c_1c_2c_3) = \frac{1}{2}$, $S'(\triangle c_3c_1c_i) = \frac{1}{2}x_i$ and $S'(\triangle c_1c_2c_i) = \frac{1}{2}y_i$. Thus $\rho_{3,1,i} = \frac{S'(\triangle c_3c_1c_i)}{S'(\triangle c_1c_2c_3)} = x_i$ and $\rho_{3,1,i} = \frac{S'(\triangle c_3c_1c_i)}{S'(\triangle c_1c_2c_3)} = y_i$. Proof complete.

Corollary 1. *Let $\triangle_\beta$ be a non-degenerate triangle and*

$$\rho_{i,\beta(1),\beta(2)} = \rho_{j,\beta(1),\beta(2)} = \rho_{k,\beta(1),\beta(2)} = \rho^*,$$

then the points a_i, a_j and a_k are collinear.

Proof. According to lemma 1 there exists an affine transform A such that $A(a_i) = c_i$, $A(a_j) = c_j$,$A(a_j) = c_j$ so that $Y(c_i) = Y(c_j) = Y(c_k) = \rho^*$. Then c_i, c_j and c_k are collinear, therefore a_i, a_j and a_k are collinear too.

Corollary 2. *Two non-degenerate images A and B are affine-equivalent if and only if their code matrices are equal for some points numeration.*

Note 2. This corollary is analogous to Theorem 1 from [4] (for another coding function).

Proof. Oriented areas ratio is conserved under affine transformation so if A is affine image of B their code matrices are the same.

Let us prove the sufficiency. In a non-degenerate image there exists a non-degenerate triangle $\triangle_\beta(A) = \triangle a_ia_ja_k$. As the code matrices are equal then the respective triangle $\triangle_\beta(B) = \triangle b_ib_jb_k$ is non-degenerate too. Consider an image C with points' coordinates $X(c_i) = \rho_{i,\beta(3),\beta(1)}$, $Y(c_i) = \rho_{i,\beta(1),\beta(2)}$). Then by lemma 1 one can construct the affine transforms $F_1 : A \to C$ и $F_2 : B \to C$. Therefore, A and B are affine equivalent. Proof complete.

Let us show by an example that properties 1–5 are not sufficient for the existence of an image, with a given code matrix.

Example 2. Consider a regular pentagon with vertices a_1, ..., a_5. Let us denote the intersection points of the diagonals b_1, ..., b_5 (see Fig. 3). Place the points m_1, m_2 and m_3 inside the triangles $\triangle a_1 a_2 b_4$, $\triangle a_4 a_5 b_2$ and $\triangle a_3 b_1 b_5$, respectively.

Let place unit masses at these points. For a triangle $\triangle a_i a_j a_k$, consider the total mass of points, located inside it. We will call this mass taken with the $+/-$ sign depending on the direction of the bypass, pseudo-area of a triangle $triangle a_i a_j a_k$ and denote $S^*(\triangle a_i a_j a_k)$. Obviously, for the pseudo-area, additivity property holds. Consider the matrix $R = (r_{\alpha\beta})$, $r_{\alpha\beta} = S^*(\triangle_\alpha)/S^*(\triangle_\beta)$. Properties 1-3 are fulfilled for it by construction, property 4 follows from the definition of pseudo-area, and property 5 is due to its additivity.

Suppose that R is code matrix for some image $a'_1, ..., a'_5$. Notice that

$$r_{123,123} = r_{124,123} = r_{125,123} = 1,$$

then by corollary 1, the points a'_3, a'_4 and a'_5 are collinear.

Then $\triangle a'_3 a'_4 a'_5$ has zero area, thus $r_{345,123} = 0$. But it contradicts to $r_{345,123} = 1 \neq 0$.

Note 3. The concept of pseudo-area can be defined more strictly. To do this, place rice. 3 on the complex plane and interpret points as elements of $\mathbb{C}$. Consider a meromorphic function $f(z) = \frac{1}{z-m_1} + \frac{1}{z-m_2} + \frac{1}{z-m_3}$, then, one can define the pseudo-area of a triangle as the following contour integral

$$S^*(\triangle a_i a_j a_k) = \frac{1}{2\pi i} \oint\limits_{\triangle a_i a_j a_k} f(z) \mathrm{dz}.$$

3. Основные результаты

So, conditions 1-5 are not sufficient for the existence of an image with the given code matrix. The theorem below answers the question — what additional conditions can ensure the existence of such an image.

Theorem 1. *Let the matrix R satisfy conditions 1-5. Let there exist $\alpha, \beta \in \mathcal{A}$ such that $r_{\alpha,\beta} \neq \infty$. Then R to is the code matrix of some non-degenerate image, if and only if for any $i, j = 1, ..., n$ the equality*

$$\rho_{\beta(1),i,j} = \rho_{i,\beta(3),\beta(1)} \cdot \rho_{j,\beta(1),\beta(2)} - \rho_{j,\beta(3),\beta(1)} \cdot \rho_{i,\beta(1),\beta(2)}, \tag{1}$$

is satisfied (here ρ_α stands for $r_{\alpha,\beta}$).

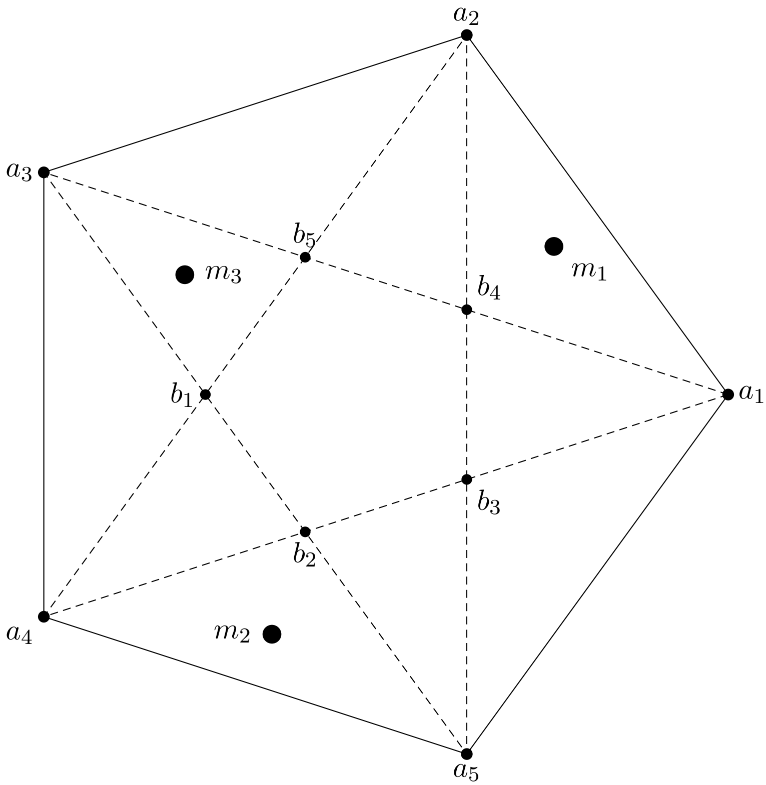

Fig. 3. Example 2.

Proof
Necessity. Let $\beta = [1,2,3]$ without loss of generality. Consider an affine transform A from lemma 1 proof. It maps the points a_s, $s = 1,2,3$ to $c_1(0,0)$, $c_2(1,0)$ and $c_3(0,1)$, respectively. Obviously $S'(\triangle c_1c_2c_3) = \frac{1}{2}$. The transform maps the points a_i and a_j to c_i and c_j, with the coordinates $X(c_i) = \rho_{i,3,1}$, $Y(c_i) = \rho_{i,1,2}$, and $X(c_j) = \rho_{j,3,1}$, $Y(c_j) = \rho_{j,1,2}$, respectively. Then oriented area of $\triangle c_1c_ic_j$ is computed by well-known formula:

$$S(\triangle c_1c_ic_j) = \frac{1}{2}det\begin{pmatrix} X(c_i) & Y(c_i) \\ X(c_j) & Y(c_j) \end{pmatrix} = \frac{1}{2}(\rho_{i,3,1}\cdot\rho_{j,1,2} - \rho_{j,3,1}\cdot\rho_{i,1,2}).$$

Divide the equality by $S'(\triangle c_1c_2c_3) = \frac{1}{2}$, we have an equality (1).

Sufficiency Let the matrix R satisfies the equality(1). Construct the set of points $\{a_i\ :\ i = 1,...,N\}$ such that coordinates are $X(a_i) = \rho_{i,3,1}$, $Y(a_i) = \rho_{i,1,2}$. Construct the code matrix for that image $R^* = (r^*_{\alpha\beta})$. Later we will show that it equals to the given matrix R.

Denote $\rho^*_\alpha = r^*_{\alpha\beta}$. Let $\alpha = [i, j, k]$, consider an intersection $P = \{i, j, k\} \cap \{1, 2, 3\}$. Actually P is a common indices set for α and β.

Possible cases:

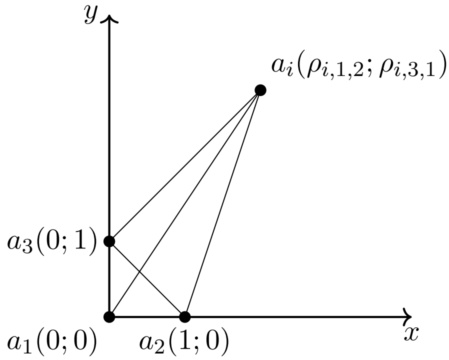

Fig. 4. Случай $|P| = 2$.

- The case $|P| = 3$. Then $\alpha = [j, i, k]$ is a permutation of $1, 2, 3$, i.e. it is eithe $\alpha = \beta$ or $\alpha = \bar{\beta}$. Then $S'(\triangle_\beta) = \frac{1}{2}$ and $S'(\triangle_{\bar{\beta}}) = -\frac{1}{2}$. Thus, $\rho^*_\beta = 1 = \rho_\beta$ and $\rho^*_{\bar{\beta}} = 1 = \rho_{\bar{\beta}}$.

- The case $|P| = 2$ and $1 \in P$ (see fig. 4). In other words the triangles $\triangle_\alpha$ and $\triangle_b$ share two common vertices and one of them is the origin of coordinates. One of i, j, k is not an element of $\{1, 2, 3\}$, let it be i without the lost of generality. If the remaining indices are 1 and 2, then[4] $\alpha = \gamma$ or $\alpha = \bar{\gamma}$, where $\gamma = [1, 2, i]$. If $\alpha = \gamma$ then

$$S'(\triangle_\gamma) = S'(\triangle a_i a_1 a_2) = \frac{1}{2} Y(a_i) = \frac{1}{2}\rho_{i,1,2}.$$

  Divide the equality by $S'(\triangle_\beta) = 1/2$ we have $\rho^*_\gamma = \rho_\gamma$. If, on the other hand $\alpha = \bar{\gamma}$, then $\rho^*_\alpha = \rho^*_{\bar{\gamma}} = -\rho^*_\gamma = -\rho_\gamma = \rho_{\bar{\gamma}}$. The case $P = \{1, 3\}$ is considered the same way.

- The case $P = \{2, 3\}$ (see fig. 4). In other words the triangles $\triangle_\alpha$ and $\triangle_b$ share two common vertices a_1 and a_2. One of i, j, k is not an element of $\{2, 3\}$, let it be i without the lost of generality.

  Then either $\alpha = \delta$ or $\alpha = \bar{\delta}$, where $\delta = [i, 3, 2]$. If $\alpha = \delta$ then by property 5 (additivity)

$$\rho^*_\alpha = \rho^*_{i,3,2} = \rho^*_{i,3,1} + \rho^*_{i,1,2} - \rho^*_{1,2,3} = \rho_{i,3,1} + \rho_{i,1,2} - \rho_{1,2,3} = \rho_{i,3,2} = \rho_\alpha.$$

[4]Recall that the multi-indices are equivalent with respect to a cyclic permutations.

The third equality here follows from $\rho^*_{i,1,2} = \rho_{i,1,2}$ and $\rho^*_{i,3,1} = \rho_{i,3,1}$ proved earlier. If, on the other hand $\alpha = \bar{\delta}$, then $\rho^*_\alpha = \rho^*_{\bar{\delta}} = -\rho^*_\delta = -\rho_\delta = \rho_{\bar{\delta}}$.

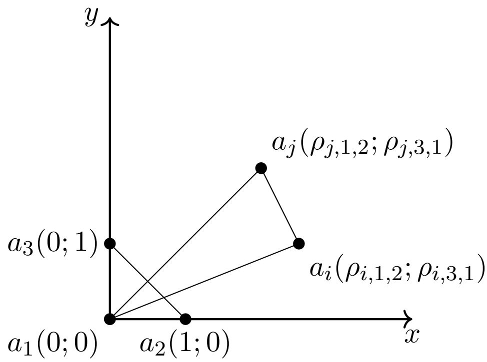

Fig. 5. The case $P = \{1\}$.

- The case $P = \{1\}$ (see fig. 5). In other words the triangles $\triangle_\alpha$ share $\triangle_\beta$ a single common vertex located in the origin.

  Assume that the rest to vertices are a_i and a_j without the generality loss. Consider the triangle $\triangle a_1 a_i a_j$ with oriented area

$$S'(\triangle a_1 a_i a_j) = \frac{1}{2} \cdot \det \begin{pmatrix} X(a_i) & Y(a_i) \\ X(a_j) & Y(a_j) \end{pmatrix} = \frac{1}{2} \cdot (\rho_{i,3,1} \cdot \rho_{j,1,2} - \rho_{j,3,1} \cdot \rho_{i,1,2}).$$

  Divide the equality by $S'(\triangle a_1 a_2 a_2) = 1/2$, we will have $\rho^*_{1,i,j} = \rho_{i,3,1} \cdot \rho_{j,1,2} - \rho_{j,3,1} \cdot \rho_{i,1,2} = \rho_{1,i,j}$, where the last equality follows from (1).

- The general case: when i, j, k are arbitrary indices. Both ρ and ρ^* are additive. So $\rho^*_{i,j,k} = \rho^*_{1,i,j} + \rho^*_{1,j,k} - \rho^*_{1,i,k}$, that (as in the previous case) equals to $\rho_{1,i,j} + \rho_{1,j,k} - \rho_{1,i,k} = \rho_{i,j,k}$. Proof complete.

Getting back to the codes using non-oriented area ([4]).

Definition 2. Let call the *sign assignment* an arbitrary set of numbers $s_{\alpha,\beta} \in \{\pm 1\}$, where $\alpha, \beta \in \mathcal{A}$. Let call the sign assignment *consistent* if for all $\alpha, \beta, \gamma \in \mathcal{A}$ the following conditions hold:

1) $s_{\alpha\beta} \cdot s_{\beta\gamma} = s_{\alpha\gamma}$;

2) $s_{\pi(\alpha)\sigma(\beta)} = (-1)^\pi \cdot (-1)^\sigma \cdot s_{\alpha\beta}$, $\pi, \sigma \in S_3$.

Note 4. Obviously, a consistent sign assignment satisfies $s_{\alpha\alpha} = 1$ and $s_{\alpha\beta} = s_{\beta\alpha}$ for all $\alpha, \beta \in \mathcal{A}$.

Corollary 3. *The set of numbers $r^*_{\alpha\beta}$ is the code of a non-degenerate image if and only if there exists a consistent sign assignment $s_{\alpha,\beta}$, such that for $r_{\alpha\beta} = s_{\alpha\beta} \cdot r^*_{\alpha\beta}$ the conditions 1–5 and (1) hold.*

Proof

Necessity Just set $s_{\alpha,\beta} = 1$ if the triangles $\triangle_\alpha$ and $\triangle_\beta$ have the same orientation. Then apply theorem 1.

Sufficiency Construct an image with code matrix $R = ((r_{\alpha\beta}))$ by theorem 1. Then take $r^*_{\alpha\beta} = |r_{\alpha\beta}|$.

4. Conclusion

The main result of this paper completely describe the set of non-degenerate images codes. The future plans are to build similar conditions for other coding functions, e.g., projective equivalence preserving coding functions or for 3-D affine equivalence preserving.

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