Cellular automata with locators [1]

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This article introduces a new mathematical object called a cellular automaton with locators. It was created by implementing new functionality for an automaton to broadcast broadcasting signals and to receive summarized broadcasting signal of all elementary automata. This article highlights several problems which solution is greatly simplified by using cellular automata with locators instead of traditional cellular automata.

Keywords: cellular automata, homogeneous structures, firing squad problem, motion picture design, constructing the shortest path.

1. Introduction

Cellular automata (other names: self-reproducing automata and homogeneous structures) are discrete mathematical models of a wide class of real systems along with the processes taking place in them.

Theory of self-reproducing automata was introduced by John von Neumann[1, 2] to describe the processes self-reproduction in biology and technology. His model was further developed and the term "Cellular automaton" as it described below was used by A. Burks [3], E. Moore [4], V. B. Kudryavtsev, A. S. Podkolzin, A. A. Bolotov [5] and other researchers.

Cellular automaton is a mathematical object with discrete space and time. Its every position in space represented by a single cell, and each moment in time represented by discrete time step or generation. The state of each spatial cell is determined by very simple rules of interaction. These rules prescribe changes in the state of each cell in the next time step in response to the current state of neighboring cells. Moreover, for different cells, the rules for changing states may be different.

If we choose a finite automaton as a transformer of information standing in a cell of space, the same one for all cells, then we come to the concept of a homogeneous structure. In this case, the cellular automaton is an infinite automaton circuit constructed as follows. Consider the k-dimensional Euclidean space. We divide it into hypercubes with a unit edge, the edges of which are parallel to the coordinate axes. In each hypercube we put the same finite automaton V with m inputs and one output. We branch the output of

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the automaton and connect it with the inputs of its neighbors in the same way for all hypercubes in space. We get an infinite homogeneous way arranged automaton scheme, which is called a cellular automaton. The sequence of states of individual automata V, containing the states of all automata of the circuit, form the state of the cellular automaton. The sequence of states of a cellular automaton arising from the synchronous operation of all its individual automata is called the functioning of a cellular automaton.

Cellular automata are a discrete mathematical model of a wide class of real systems along with processes occurring in them, such as physical media in which thermal and wave phenomena are realized, chemical solutions with reactions in them, biological tissues in which metabolism occurs, and technical control schemes processing mechanical and electrical signals, computational circuits, etc.

If we set the initial states of the automata, then in the circuit the states of the automata start to change in the way determined by the laws of the functioning of automata and the relationships between them. The phenomenon of a global change in these states is the main object of study in the theory of cellular automata.

This article introduces the generalization of a cellular automaton, which is proposed to be called a cellular automaton with locators.

One of the serious limitations of cellular automata is the limitedness of the neighborhood pattern, i.e. each automaton can see a certain number of its neighbors, and thus signals in cellular automata propagate relatively slowly. It is proposed to provide cellular automata with the ability to transmit some signals to all elementary automata at the same time, which will overcome the locality property.

Here we can recall the model of an incompressible fluid, in which the signals also instantly propagate throughout the entire volume. A similar picture is observed in quantum mechanics and in quantum cellular automata [6], when a change in the state of one automaton causes a change in the state of all automata "entangled" with it. In the work [7], the concept of nonlocal cellular automata is introduced. In this paper, nonlocality means that for each elementary automaton, the set of its neighbors is chosen randomly, and thus, elementary automata that are far apart from each other can be neighboring.

In real life, a person, when he wants to transmit information not only to visible neighbors, he can take advantage of such techniques as the supply of light signals using signal flares. An even more common method is the use of radio and television broadcasts.

Here, we also introduce the concept of broadcasting. Each elementary automaton is considered to be able to broadcast some signal from the finite alphabet on the air. Elements of the alphabet form a finite additive commutative semigroup, and the air itself is a potentially infinite adder of signals of elementary automata, where the defining operation of this semigroup acts as a sum. At the next clock, each elementary automaton receives a total signal from the air and changes its state according to the signal. In nature, such an adder is air that sums all the radio signals in a natural way, and in fact each of the receivers gets the same signal at the input, and only then it extracts the necessary component from the general signal.

With the help of this principle, one can implement a new type of integrated circuits that use some substrate as an adder, onto which all elementary automata will dump some switching or emergency signals.

Introduction of broadcasting concept and the ability to transmit signals instantly at any distance allows one elementary automaton to control the behavior of another elementary automaton arbitrarily far from it. We consider cellular automata with locators that can receive signals from certain directions. In other words, each elementary automaton has several locators directed in different directions, and it can use these locators to receive signals from the very directions.

This article introduces a formal model of cellular automata with locators. The solution of several tranditional problems and new challenges using standard cellular automata is given. Then, it is shown that the same problems can be solved much easier using cellular automata with locators.

2. The concept of cellular automaton with locators

We introduce the concept of a cellular automaton with locators based on the definition of a cellular automaton from [8].

By a *solid angle* in $\mathbb{R}^k$ we mean the union of all the rays in the space $\mathbb{R}^k$ emanating from a given point (*vertex of an angle*) and intersecting some hypersurface in $\mathbb{R}^k$. We assume that a solid angle does not contain its vertex. In particular, in this paper we consider two degenerate cases: the full solid angle coinciding with $\mathbb{R}^k$ without the vertex of the angle, which we denote by Ω, and solid angles equal to one ray. If a solid angle is a ray, we denote it by a vector defining its direction.

A *cellular automaton with locators* is a 8-tuple

$$\sigma = (\mathbb{Z}^k, E_n, V, E_q, +, L, \varphi, \psi)$$

where $\mathbb{Z}^k$ is the set of k-dimensional vectors with integer coordinates, $E_n = \{0, 1, \dots, n-1\}$, $V = (\alpha_1, \dots, \alpha_{h-1})$ is an ordered set of pairwise different nonzero vectors from $\mathbb{Z}^k$, $E_q = \{0, 1, \dots, q-1\}$, $+$ is a commutative semigroup operation defined on E_q, $L = (\nu_1, \dots, \nu_m)$ is an ordered set of pairwise different solid angles in $\mathbb{R}^k$ with a vertex at the origin, $\varphi : E_n^h \times E_q^m \to$

E_n is a function depending on the variables $x_0, x_1, \ldots, x_{h-1}, z_1, \ldots, z_m$ such that $\varphi(0, \ldots, 0) = 0$, $\psi : E_n^h \times E_q^m \to E_q$ is a function that depends on the variables $x_0, x_1, \ldots, x_{h-1}, z_1, \ldots, z_m$. Here the variables $x_0, x_1, \ldots, x_{h-1}$ take values from E_n and the variables $z_1, \ldots, z_m$ take values from E_q. Elements of the set $\mathbb{Z}^k$ are called *cells* of the cellular automaton σ; elements of the set E_n are called *cell states* of the cellular automaton σ; the set V is called the *neighborhood pattern* of the cellular automaton σ; elements of the set E_q are called *broadcasting signals*; the set L is called the *locator pattern* of the cellular automaton σ; the function φ is called the *local transition function* of the automaton σ; the function ψ is called the *broadcasting function* of the automaton σ. The state 0 is interpreted as *quiescent state* and the condition $\varphi(0, \ldots, 0) = 0$ is interpreted as a condition for maintaining the quiescent state.

Here we need to introduce an ordering of the neighborhood pattern V and the locator pattern L in order to establish a one-to-one correspondence between vectors from V and solid angles from L and variables x_0, $x_1, \ldots, x_{h-1}$, $z_1, \ldots, z_m$ of the local transition function φ and the broadcasting function ψ respectively. We can make this correspondence more explicit if we index the variables of the functions φ and ψ by the vectors and solid angles themselves, i.e. assume that the local transition function φ and the broadcasting function ψ depend on the variables $x_{\mathbf{0}}$, $x_{\alpha_1}, \ldots, x_{\alpha_{h-1}}$, $z_{\nu_1}, \ldots, z_{\nu_m}$, where the index of the first variable is the zero vector $\mathbf{0} = (0, \ldots, 0) \in \mathbb{Z}^k$. If we index the variables of the local transition function and broadcasting function in this way, we can write them in any order, and then we can define the neighborhood pattern and the locator pattern simply as a set, not an ordered set.

In the quiescent of this section we use these conventions: consider the neighborhood pattern as a set of vectors, and the locator pattern as a set of solid angles and index the variables of the local transition function and broadcasting function by the vectors from the neighborhood pattern and solid angles from the locator pattern. At the same time, we often omit the outer parentheses of the vectors in the indices. For example, if $k = 2$, $n = 2$, $q = 2$, $V = \{(-1, 0), (1, 0)\}$, and $L = \{\Omega, (0, 1)\}$, then a local transition function may look like this: $\varphi = x_{-1,0} \& z_\Omega \vee x_{1,0} \& z_{0,1}$.

If $\alpha \in \mathbb{Z}^k$, ν is a solid angle with vertex at the origin, then we denote by $\nu(\alpha)$ the solid angle obtained by translation of the angle ν to the point α.

If $\alpha \in \mathbb{Z}^k$ is a cell of a cellular automaton with locators σ, then the set $V(\alpha) = \{\alpha, \alpha + \alpha_1, \ldots, \alpha + \alpha_{h-1}\}$ is called the *neighborhood of the cell* α, and elements of the set $L(\alpha) = \{\nu_1(\alpha), \ldots, \nu_m(\alpha_m)\}$ are called *locators of the cell* α.

A *state of a cellular automaton with locators* σ is a pair (e, f), where e is an arbitrary function from the set $\mathbb{Z}^k$ to the set E_q, called *broadcast state*, f

is an arbitrary function from the set $\mathbb{Z}^k$ to the set E_n and called *distribution of states of the cellular automaton with locators* σ. Such a function can be interpreted as a certain mosaic arising in the k-dimensional space as a result of assigning a certain state from the set E_n and some signal from the set E_q to each point with integer coordinates. The set of all possible states of a cellular automaton with locators is denoted by Σ.

If $\alpha \in \mathbb{Z}^k$ and (e, f) is a state of a cellular automaton with locators σ, then the value $e(\alpha)$ is called *the signal of the cell* α*, defined by the state* (e, f), and the value $f(\alpha)$ is *the state of the cell* α*, determined by the state* (e, f). For each $i \in \{1, \ldots, m\}$ the value

$$s_i(\alpha) = \sum_{\beta \in \nu_i(\alpha) \cap \mathbb{Z}^k} e(\beta) \tag{1}$$

we call *the value of the locator* ν_i*, determined by the state* (e, f). Here, the semigroup operation $+$ defined on E_q is used to sum signals.

On the set Σ we define the *global transition function* Φ of a cellular automaton with locators σ, putting $\Phi(e, f) = (e', f')$, where $(e, f), (e', f') \in \Sigma$ and for any cell $\alpha \in \mathbb{Z}^k$ the following identities hold

$$f'(\alpha) = \varphi(f(\alpha), f(\alpha + \alpha_1), \ldots, f(\alpha + \alpha_{h-1}), s_1(\alpha), \ldots, s_m(\alpha)), \tag{2}$$

$$e'(\alpha) = \psi(f(\alpha), f(\alpha + \alpha_1), \ldots, f(\alpha + \alpha_{h-1}), s_1(\alpha), \ldots, s_m(\alpha)). \tag{3}$$

A meaningful interpretation of the mapping Φ is that the signal of each cell and the state of each cell “after the transition” is determined by the state of the neighborhood of the cell and by the values of the locators “before the transition” using the rules ψ and φ in the same way for all cells.

By the *behavior of a cellular automaton with locators* σ we call a sequence $(e_0, f_0), (e_1, f_1), (e_2, f_2), \ldots$ of states such that the equation $(e_{i+1}, f_{i+1}) = \Phi(e_i, f_i)$ holds for all $i = 0, 1, 2, \ldots$. The state (e_i, f_i) is called the *state of the cellular automaton with locators* σ *at the time* i, and (e_0, f_0) is also called the *initial state of the cellular automaton with locators* σ.

A state of a cellular automaton is called a *configuration* if only a finite number of cells are in a state other than 0 and the signals of all the cells are zero. The set of configurations is denoted by Σ'.

If a certain state of a cellular automaton is specified, then cells that are in a state other than 0 are called *active*.

Furher, we demonstrate several problems for cellular automata and show how their solutions are significantly simplified in case of using cellular automata with locators.

3. A firing squad problem

A firing squad problem was first proposed by J. Mayhill in 1957 and published (with a solution) in 1968 by F. Moore [9].

In this problem, we consider a one-dimensional cellular automaton on ^{1}. The neighborhood pattern is $V = \{-1, 1\}$, i.e. each cell has two neighbors left and right. The set of states have at least three states: 0 — quiescent state, 1 — soldier in initial state, 2 — fire. There is a restriction on the transition function that a soldier in the initial state, having neighbors of such soldiers, does not change the state, i.e. $\varphi(1, 1, 1) = 1$. The initial configuration is a continuous segment of r cells in state 1 (soldiers), and all other cells are in quiescent state 0. It is necessary that at some point in time all active cells switch to the state 2 at the same time (fired).

The standard solution to the above problem contains two waves of states propagating through a number of soldiers, one of which moves three times faster than the other one. The faster wave is reflected from the far edge of the row and meets the slower one in the center. After that, two waves are divided into four waves, moving in different directions from the center. The process continues, each time doubling the number of waves, until the length of the segments of the row becomes equal to 1. At this moment, all the soldiers shoot. This solution requires $3r$ time units for r soldiers.

The cellular automaton with the locators σ_0, which solves the firing squad problem, has the following form $\sigma_0 = (^1, E_3, V = \{-1, 1\}, E_2, \vee, L = \{\Omega\}, \varphi, \psi)$, where $\vee$ is the disjunction taken as the determining operation on broadcasting function ψ takes the value 1 only for the leftmost soldier in the initial state, i.e. $\psi(x_0, x_{-1}, x_1, z_\Omega) = x_0 \bar{x}_{-1} \bar{z}_\Omega$, the local transition function takes the value 2 only if the cell is in state 1 and the broadcasting signal is 1, and does not change state in all other cases, i.e.

$$\varphi(x_0, x_{-1}, x_1, z_\Omega) = \max(2 \cdot ((x_0 = 1) \& (z_\Omega = 1) \vee (x_0 = 2)), 1 \cdot (x_0 = 1)).$$

Hence, the firing squad problem can be solved in 2 clocks using three states and two broadcasting signals.

Note that the described solution is fully consistent with the real-life protocol used by the military: the commander (the leftmost soldier) commands “fire” and the whole squad shoots.

4. Unidirectional movement of a point on the ray

The problem of unidirectional motion of a point on the ray, proposed and solved in the paper [10] by E.E. Titova, is as follows.

The set of cells is a set of natural numbers, i.e. ray is directed to the right. Each cell has two neighbors, one on the left and one on the right of itself. Some of the states of the cells are called labels and considered black, other states are considered white. Configurations are considered to be correct when there is exactly one black cell (called a point) on the ray. The cell corresponding to the number 1 (the leftmost cell) does not have a neighbor on the left of itself, and we will consider the variable corresponding to the state of the neighbor on the left of this cell as a control input, to which we can apply any control actions. The described set of cells with one control input will be called the *screen*.

Formally, the *screen* is the cellular automaton $S = (, E_n, V = \{-1, 1\}, \varphi, M)$, where n is the number of states of the cell of the cellular automaton, and $\varphi : E_n^3 \to E_n$ is a local transition function, M is a set of labels, $M \subset E_n$, $0 \notin M$. If the cell is in a state of M, then informally we believe that it is painted black, otherwise it is painted white. We call the variable x_{-1} of the local transition function of the cell corresponding to the number 1, (of the *leftmost cell) the control input of the screen S.*

Law of motion is an infinite sequence (superword) of zeros and ones. If $F = f_1, f_2, f_3, \ldots$ is the law of motion, then we denote by $F(t)$ the t-th element of the sequence, i.e. $F(t) = f_t$.

We say that *on the screen S the point moves according to the law F*, if the following conditions are satisfied:

1) at some point in time, a label appears in the leftmost cell of the screen (before that there are no labels on the screen), this moment is called *the movement start*;

2) changing the position of the label on the screen at the t-th moment from the movement start corresponds to the t-th letter in the superword F, namely, if $F(t) = 0$, then in $(t+1)$-th moment the label remains in the same cell where it was at the current moment, if $F(t) = 1$, then in $(t+1)$-th moment the label moves one cell to the right, compared to its current position;

3) at each moment of time after the movement start, there is exactly one label on the screen.

A S screen will be called *universal for the set of laws of motion $\mathcal{F}$* if for any F from $\mathcal{F}$ there is such a control sequence supplied to the control input of the screen that provides a point movement by the law F on the screen.

We denote by $\mathcal{F}^s$ the set of such laws of motion F that do not contain more than s ones in a row.

The following theorems are proved in [10].

Theorem 1. *For any screen S, there exists a law of motion $F \in \{0,1\}^\infty$ such that it is impossible to realize the motion of a point according to the law F on the screen S.*

Theorem 2. *There is a law of motion $F \in \{0,1\}^\infty$, the movement of which cannot be realized on any screen S.*

Theorem 3. *There is a universal screen with $2s+2$ states for the set of laws of motion $\mathcal{F}^s$.*

The question of describing the set of all realized laws of motion remains open, although G.V. Kalachev and E.E. Titova [11] have significantly advanced in this direction.

We present a cellular automaton with locators that solves the problem of unidirectional motion of a point on the ray.

Consider the following cellular automaton with the locators $\sigma_1 = (, E_2, V = \{-1, 1\}, E_2, \vee, L = \{\Omega\}, \varphi, \psi, M = \{1\})$, where $\vee$ is the disjunction taken as the determining operation on the semigroup of signals $E_2 = \{0, 1\}$, the locator pattern consists of one full solid angle Ω, the set of labels consists of one character 1, broadcasting function ψ is identically zero, the local transition function takes the value 1 in only two cases: if the cell is in state 1 and the broadcasting signal is 0, or if the cell on the left is in state 1 and the broadcasting signal is 1, i.e. $\varphi(x_0, x_{-1}, x_1, z_\Omega) = x_0 \& \bar{z}_\Omega \vee x_{-1} \& z_\Omega$.

We assume that the variable x_{-1} of the local transition function of the leftmost cell is the control input. In addition, we will consider that signals from E_2 can be broadcasted as control actions.

It is clear that in order to start moving, you need to send 1 to the control input, as well as send 1 to the air. As a result, the label appears on the screen in the leftmost cell. Further, in order to realize the law of motion F, it is necessary to send the value $F(t)$ to the air at the time t.

Thus, using cellular automata with locators, any law of motion can be realized, and the number of states of this automaton is 2, and the cardinaity of the broadcasing alphabet is 2.

In fact, with the help of signals on the air we give commands to the point, move it or stand.

5. Construction of the shortest path

The problem of constructing the shortest path for cellular automata is as follows. In the initial configuration, there are only two cells in the active state, which we will call the starting points. The shortest path is considered to be built if, at some point in time, the configuration becomes stable, and the active cells of this configuration form the shortest path between the starting points.

An adaptation of the traditional way to solve this problem to cellular automata is provided in [12]. Such adaptation involves the presence of three stages:

1) A propagation of an expanding signal from one of the starting points. When expanding, each cell remembers where the signal came from. This will allow to carry out a reverse move later.

2) When the wave reaches the second starting point, a reverse movement is carried out, leading to the first point, which gives the shortest path.

3) At the same time, a purification wave starts. This wave switches all the cells except the path cells into the quiescent state. In order for this wave to catch up with the expanding wave, the expanding wave from the first stage must expand at a half speed, and the purification wave must expand at a unit speed.

The work [12] gives a proof that the automaton proposed by the authors has 14 states. This work does not estimate the time it takes to build the path, but it is not difficult to see that the time to build the path is no less than $6n$, where n is the Manhattan distance between the starting points. Here, $2n$ clock cycles are used at the first stage, and $4n$ cycles are necessary for the third stage. It is possible to speed up the process by launching an expanding wave from both starting points, but it is clear that the path construction time will be proportional to the distance between the starting points.

Now consider the solution to the problem by cellular automata with locators.

Consider the following cellular automaton with locators $\sigma_2 = ({}^2, E_2, V = \{(-1,0),(0,1),(1,0),(0,-1)\}, E_2, \vee, L = \{(-1,0),(0,1),(1,0),(0,-1)\}, \varphi, \psi)$, where $\vee$ — a disjunction taken as a determining operation on a semigroup of signals $E_2 = \{0,1\}$, a neighborhood pattern is "cross", a pattern of locators consists of four rays directed left, up, right and down, broadcasting function ψ takes the value 1 if the cell is in state 1 and one of four cases occurs: if all its neighbors are in state 0; the cell does not have a neighbor from above in state 1 and the upper locator receives signal 1; the cell does not have a left neighbor in state 1 and the left locator receives signal 1; the cell has no neighbor to the right in state 1 and the right locator receives signal 1; i.e.

$$\psi(x_0, x_{-1,0}, x_{0,1}, x_{1,0}, x_{0,-1}, z_{-1,0}, z_{0,1}, z_{1,0}, z_{0,-1}) = \\ = x_0(\bar{x}_{-1,0}\bar{x}_{0,1}\bar{x}_{1,0}\bar{x}_{0,-1} \vee \bar{x}_{0,1}z_{0,1} \vee \bar{x}_{-1,0}z_{-1,0} \vee \bar{x}_{1,0}z_{1,0});$$

the local transition function takes on value 1 if the cell was in state 1, or if signals from the air came to one of four pairs of locators at the same time: top and right, top and left, top and bottom, left and right, i.e.

$$\varphi(x_0, x_{-1,0}, x_{0,1}, x_{1,0}, x_{0,-1}, z_{-1,0}, z_{0,1}, z_{1,0}, z_{0,-1}) = \\ = x_0 \vee z_{0,1}z_{1,0} \vee z_{0,1}z_{-1,0} \vee z_{0,1}z_{0,-1} \vee z_{-1,0}z_{1,0}.$$

We show that the aforementioned cellular automaton with locators solves the problem of constructing the shortest path.

In the initial (zero) clock cycle on the plane, there are only two active cells, which we call the initial ones.

We consider various cases of the location of the initial cells.

Case 1. The initial cells are located on the same horizontal. We denote by A the left initial cell, and by B the right one.

Case 1.1. If the initial cells are adjacent, then this pair of cells makes up the shortest path. It remains to note that the signals will not be broadcasted on the air, therefore, new active cells will not appear, and, therefore, the configuration will remain stable.

Case 1.2. If the initial cells are not adjacent, then the broadcasting function of each of the initial cells will become equal to 1, since these cells have no neighbors. Consequently, on step 1, a signal will be broadcast from A and B cells, and for all cells between A and B cells, the left and right locators will receive signals. Therefore, the local transition functions of these cells will take the value 1. Therefore, on step 2, all cells between A and B will go to state 1. The shortest path is constructed. Since all cells have neighbors, the broadcasting function of all cells will take the value 0, and the resulting configuration will remain stable.

Case 2. Initial cells are located on one vertical. This case is proved similarly to the case 1.

Case 3. The initial cells are in general position. Let's mentally draw vertical and horizontal lines through the initial cells. As a result, we get an imaginary rectangle with sides parallel to the coordinate axes, at the two diagonal vertices of which the initial cells are located.

Case 3.1. One initial cell is located in the upper left corner of the rectangle (we denote it by A), and the second initial cell is located in the lower right corner (we denote it by B).

Since the initial cells have no neighbors, the broadcasting function of these cells will take the value 1. Therefore, on step 1, a signal will go from the cells A and B on the air. The cell located in the lower left corner of the rectangle (denoted by C) will receive signals from the upper and right locators. Therefore, its local transition function will take the value 1. Therefore, at step 2, the cell C will become active.

Case 3.1.1. The cell C is adjacent to both A and B. Therefore, the shortest path is built. All cells have neighbors, therefore, signals will not be broadcasted anymore, and the configuration will remain stable.

Case 3.1.2. The cell C is not adjacent to A, and is adjacent to B. Then the broadcasting function of the cell C will take the value 1. Therefore, on step 3, signals from two cells will go on the air: A and C. This means that all cells between A and C will receive signals to their upper and lower locators. As a consequence, their local transition function will take the value 1. Hence, on step 4, all cells between A and C will become active. Now all cells have neighbors, therefore, signals will not be broadcasted anymore, and the configuration will remain stable.

Case 3.1.3. The cell C is adjacent to A, and is not adjacent to B. This case is proved similarly to the case 3.1.2.

Case 3.1.4. The cell C is adjacent to neither A nor B. Then the broadcasting function of the cell C will take the value 1. Thus, on step 3, signals from three cells will go on the air: A, B and C. This means that all cells between A and C will receive signals to their upper and lower locators, and all cells between B and C will receive signals to their left and right locators. Thereby, the local transition function of all these cells will take the value 1. Then, on step 4, all cells between A and C and all cells between B and C will become active. The shortest path will be built. All cells have neighbors, with the result that signals will not be broadcasted anymore, and the configuration will remain stable.

Case 3.2. One initial cell is located in the lower left corner of the imaginary rectangle, and the second initial cell is located in the upper right one. The proof is similar to the proof of the case 3.1.

Finally, we have shown that the proposed cellular automaton with locators allows us to build the shortest path in no more than 4 clocks. Moreover, it has only 2 states and 2 broadcasting signals.

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